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Non-Linear Programming Approach for Optimization of Construction Project's Control System

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Abstract

This paper discuss about the Time-Cost, Quality-Cost, Time-Quality aim optimization models of construction project. The optimization was performed by the nonlinear programming approach, NLP. Accordingly, a NLP optimization model for the cost optimization of project schedules was developed and applied. The nonlinear objective function of the total project costs was subjected to a rigorous system of generalized precedence relationship constraints between project activities, the activity duration constraints and the project duration constraints. The results of the optimization include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities. A numerical example presented at the end of the paper demonstrates the advantages of the proposed approach.

Keywords: Cost Optimization, Project Scheduling, Construction Management, Nonlinear Programming, NLP.

INTRODUCTION

One of the most important aspects of the construction project management is the cost effective scheduling. Usually used methods for the cost effective project scheduling in construction management include either the Critical Path Method (CPM) or the Program Evaluation and Review Technique (PERT) combined with trial-and-error procedure. In this way, the cost effective project schedules are achieved in a time-consuming cost-duration analysis of various alternatives for start times and durations of construction project activities. However, doubts always exists as to whether or not the obtained project schedule is optimal. To surmount the mentioned disadvantages, various different optimization methods have been proposed for the cost optimization of project schedules.

Considering the exact mathematical programming methods, the cost optimization of project schedules has been handled mainly by different linear programming (LP) methods, see e.g. Demeulemeester et al., (1998); Achuthan and Hardjawidjaja (2001); Möhring et al. (2001); Vanhoucke et al. (2002). Since the LP methods can handle only linear relations between the variables, the nonlinear terms of the optimization models have been formulated as the discrete relationships between the variables or they were approximated with (piece-wise) linear functions. However, even the earliest studies in this field have recognized the nonlinear nature of the project cost-duration relationships. Therefore, the nonlinear programming (NLP) techniques have been proposed to solve project scheduling optimization problems with nonlinear cost functions, see e.g. Kapur (1973); Deckro et al. (1995) and Turnquist and Nozick (2004). Nevertheless, in most of the published works the cost optimization of project schedules was performed considering only the Finish-to-Start precedence relationships between activities.

This paper will discuss about the cost optimization of construction project schedules performed by the NLP approach. Accordingly, a NLP optimization model for the cost optimization of project schedules was developed and applied. The nonlinear objective function of the total project costs was subjected to a rigorous system of generalized precedence relationship constraints between project activities, the activity duration constraints and the project duration constraints. The results of the optimization include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities. Anumerical example presented at the end of the paper demonstrates the advantages of the proposed approach.

NLP-MODEL FORMULATION

The cost optimization of the project schedules was performed by the NLP approach. In this way, the NLP model formulation consists of the total cost objective function, the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. The following total project cost objective function is defined for the cost optimization of project schedules:

$$C_T = \sum_{i=I} C_i(D_i) + C_I(D_P)$$
$$+P(D_L)B(D_E) \qquad (1)$$

where objective variable C_T represents the total project cost, set Icomprises the project activities i, $i \in I$, $C_i(D_i)$ denotes the direct cost-duration functions of the project activities $i, i \in I$, $C_I(D_P)$ is the project indirect cost-duration function, $P(DL)$ is the penalty-duration function and $B(DE)$ is the bonus-duration function. The variables *Di*, *DP*, *DL* and *DE* denote the durations of the project activities $i, i \in I$, the actual project duration, the amount of time the project is late, and the amount of the time the project is early, respectively. The total project cost objective function is subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints.

Each project\ activity $i, i \in I$, is connectedwith its succeeding activities j, $j \in J$ byfulfilling at least one of the followinggeneralized precedence relationship constraints:

Finish-to-Start :$S_i + D_i + L_{i,j} \leq S_j$ (2)

Start-to-Start :$S_i + L_{i,j} \leq S_j$ (3)

Start-to-Finish :$S_i + L_{i,j} \leq S_j + D_j$ (4)

Finish-to-Finish : $S_i + D_i + L_{i,j} \leq S_j + D_j$ (5)

where *Si* is the start time of activity *i*, $i \in I$, *Di* is the activity duration, *Li,j* is the lag/lead timebetween activity *i*, and the succeeding activity *j*, $j \in J$, and *Sj* is the start time of the succeeding activity *j*.

The actual project duration D_P is determined as follows:

$$D_P = S_{i\omega} + D_{i\omega} - S_{i\alpha} \qquad (6)$$

where $S_{i\omega}$ and $D_{i\omega}$ represent the start timeand the duration of the last project activity $i\omega$, and $S_{i\alpha}$ denotes the start time of the first project activity$i\alpha$.

Since the project activities must be executed between the project start and finishing time, the following constraint is set to bound the completion times of the project activities:

$$S_i + D_i - S_{i\alpha} \leq D_P \qquad (7)$$

The relationship between the actual project duration *DP*, the amount of time the project is late

DL, the amount of time the project is early *DE* and the target project duration *DT* is formulated

as follows:

$$D_P - D_L + D_E = D_T \qquad (8)$$

Only one of the variables *DL* and *DE* can, at the most, take a nonzero value in any project scheduling solution. In this way, these two variables are additionally constrained by the following equation:

$$D_L \cdot D_E = 0 \qquad (9)$$

THE OPTIMIZED QUALITY-COST CONTROL MODEL

Project quality is determined by the quality of working procedures in the construction process. The quality of each procedure will directly or indirectly affect the project quality at last. Therefore, the procedure quality is the most fundamental part for the project quality. In general, if the project time is shortened, the project quality will become poor. But many spot managers do not think so. In their opinion, even if the time is shortened, the procedure quality

does not necessarily become poor.

Any procedure needs time. The procedure quality is formed during the time. Different time contributes to different procedure quality. Here, we make an assumption that if the construction factors do not change, there is a linear function relationship between the procedure quality and its time. In other words, there is positive correlation between them. See to the figure 1 as follow.

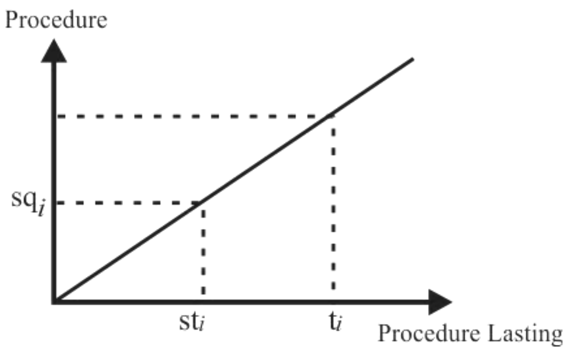

Figure 1: The Relationship between Procedure Time and Procedure Quality.

Use the continuous numbers from 0 to 1 to reflect the strictness of requirements for procedure quality. The shorter the time is, the nearer to 0 the number is, the lower the strictness of requirements for procedure quality is, and the poorer the procedure quality is. Conversely, the nearer to 1 the number is, the higher the strictness of requirements for quality, and the higher the procedure quality is.

With the assumption above, the slope of the curve, namely the Time-Quality function, is:

$$\alpha_i = \frac{nq_i - sq_i}{nt_i - st_i} \qquad (10)$$

Then, the real quality of the procedure i is:

$$Q_i = sq_i + \alpha_i(t_i - st_i) \qquad (11)$$

If the whole project includes m procedures, the whole project quality is equal to the average of weighted quality of every procedure.

$$Q = \sum_{i=I}^{m} \omega_i Q_i , \qquad \left(\sum_{i=I}^{m} \omega_i = 1\right) \qquad (12)$$

The control model of Quality-Time is:

$$\max Q = \max \sum_{i=I}^{m} \omega_i Q_i$$

$$= \max \sum_{i=I}^{m} \omega_i [sq_i + \alpha_i(t_i - st_i)]$$

Subject to :

$$nt_i \geq t_i \geq st_i > 0; \qquad (13)$$

$$\omega_k > 0, \sum_{k}^{m} \omega_k = 1$$

Here, nt_i refers to the time consumed by the procedure under the normal condition whilest_i refers to the time consumedby the procedure in order to catch up with the plan. t_i refers to the time consumed by the procedure in fact andω_i refers to the weighed quality of procedure i to the whole project quality.

The Optimized Cost-Time Control Model

The relationship between procedure time and cost is changeable under different conditions. In general, as the procedure time is shortened, the cost will rise. The shorter the time, the fast the cost increases (Xiaolin Cao. & BingHan, 2002). Of course, some procedures may adjust the operation time by increasing or re-allocating resources. Under this condition, the procedure cost does not change. But for common procedure, the relationship between its time and cost can be reflected by figure 2 as follow.

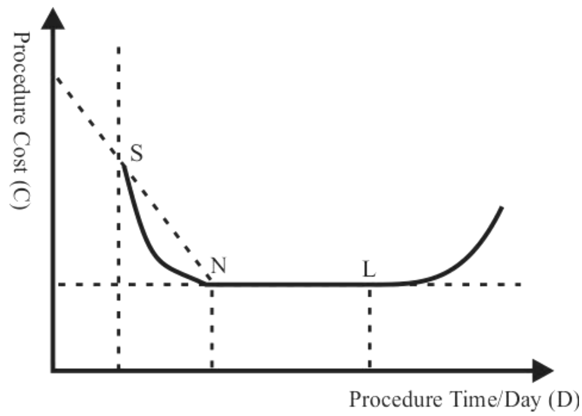

Figure 2: The Time-Cost Curve

According to the figure 2 above, the reasonable procedure time should be confined to [Ds, DL]. Without increasing procedure cost, the shorter the procedure time, the better. Because the project time is determined by the key procedure time, the key procedure time should be confined to [Ds, DN] in cost optimization. For non-key procedures, the procedure time can be adjusted between [Ds, DL] based on the requirements for resource balance as the cost optimization is over. Besides, because the Time-Cost relationship is a curve, it is complicate to establish all the relationship between every procedure cost and time. In practice, we usually replace the curve with the line from S toN. By this way, the calculated procedure cost is slightly higher than real procedure cost, what is allowable in practice. In addition, considering all uncertain information in construction, such as labors, materials, technologies, and management levels, the conservative calculation is necessary. The slope of this line is the rate of direct cost, namely the average increasing procedure cost as the procedure time is shortened by one unit. It is β_i in the followingequation.

$$\beta_i = \frac{nc_i - sc_i}{nt_i - st_i} \qquad (14)$$

In this equation, nt_i refers to the time of procedure i under the normal condition. st_i refers to the time of procedure I under the rush-up condition. nc_i refers to the cost of procedure i under the normal condition. sc_i refers to the cost ofprocedure i under the rush-up condition.

The relationship between the procedure cost and the procedure time is:

$$c_i = sc_i + \beta_i(t_i - st_i) \qquad (15)$$

Here, t_i refers to the time of procedure i. Whilec_i refers to the cost of procedure i as it consumes the time t_i.The figure 2 can not reflect the indirect cost. In optimization, we can use the rate of indirect cost to deal with the indirect cost. The so-called rate of indirect cost means the decreasing (or increasing) indirect cost as the procedure time is shortened (or prolonged) by one unit. Use to represent it. The rate of indirect cost is usually based on real facts.

NUMERICAL EXAMPLE

In order to present the applicability of the proposed NLP approach, this paper presents a numerical example of the cost optimization of the construction project schedule. The considered construction project scheduling optimization problem is a variant of the time-cost trade-off problem for a small building project presented by Yang (2005).

The construction project consists of 7 activities. The precedence relationships and the lag times between succeeding project activities are presented in Table 1. The crash/normal points and the direct cost-duration functions of the construction project activities are presented in Table 2

Table 1: Precedence Relationships and Lag Times between the Project Activities

Activity		Succeeding Activity	Precedence Relationship	Lag Time
ID	Description	ID		
1	Underground Service	2	Start-to-Start	2
2	Concrete Works	3	Finish-to-Start	3
3	Exterior Walls	4	Finish-to-Start	0
4	Roof Construction	5	Finish-to-Start	0
		6	Finish-to-Start	0
5	Floor Finishing	7	Finish-to-Start	0
6	Ceiling	7	Finish-to-Finish	6
7	Finish Work	-	-	-

Table 2: Crash/Normal Points and Direct Cost-Duration Functions of the Project Activities

Activity		Duration (Days)		Direct Cost (Rp)		
ID	Description	Crash	Normal	Crash	Normal	Direct Cost-Duration Function
1.	Underground Service	3	6	4.500	1.500	$250D_1^2 - 3.250D_1 + 12.000$
2.	Concrete Work	10	12	7.000	5.000	$-10.969{,}6\ln(D_2) + 32.258{,}5$
3.	Exterior Walls	8	12	3.600	2.000	$11.664\exp(-0.1469D_3)$
4.	Roof Construction	6	8	3.100	2.000	$-550D_4 + 6.400$
5.	Floor Finishing	3	4	3.000	2.000	$-1000D_5 + 6000$
6.	Ceiling	4	6	4.000	2.500	$-750D_6 + 7000$
7.	Finish Work	10	14	2.800	1.000	$75D_7^2 - 2.250D_7 + 17.800$
Project		42	55	28.000	16.000	

The daily indirect cost is Rp. 200.00. While the per-period penalty for late project completion is set to be Rp.400/day, the per-period bonus for early project completion is determined to be Rp.300/day. The targeted project duration is 47 days.

The objective of the optimization is to find a construction project schedule with the optimal activity start times and durations so as to minimize total project cost, subjected to the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints

The proposed NLP optimization model formulation was applied. A high-level language GAMS (General Algebraic

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Modeling System) (Brooke et al., 1988) was used for modeling and for data inputs/outputs. CONOPT (generalized reduced-gradient method) (Drud, 1994) was used for the optimization.

Since the NLP denotes the continuous optimization technique, the optimization of the project schedule was performed in two successive steps. In the first step, the ordinary NLP optimization was performed to calculate the optimal continuous variables (e.g. start times, durations, etc.) inside their upper and lower bounds, see Table 3.

Table 3: Optimum Continuous Solution

Activity	Start Time (Day)	Duration (Days)	Direct Cost (Rp)
1. Underground Service	1,000	6,000	1.500
2. Concrete Work	3,000	12,000	5.000
3. Exterior Walls	18,000	8,000	3.600
4. Roof Construction	26,000	6,667	2.733,33
5. Floor Finishing	32,667	4,000	2.000
6. Ceiling	32,667	6,000	2.500
7. Finish Work	36,667	11,333	1.933,33
		Indirect Cost (Rp)	9.400,00
		Penalty (Rp)	0.00
		Bonus (Rp)	0.00
		Total Project Bonus (Rp)	28.666,66

Table 4: Optimum Rounded Solution

Activity	Start Time (Day)	Duration (Days)	Direct Cost (Rp)
1. Underground Service	1,000	6,000	1.500
2. Concrete Work	3,000	12,000	5.000
3. Exterior Walls	18,000	8,000	3.600
4. Roof Construction	26,000	6,667	2.550,00
5. Floor Finishing	33,000	4,000	2.000
6. Ceiling	33,000	6,000	2.500
7. Finish Work	37,000	12,000	1.600,00
		Indirect Cost (Rp)	9.600,00
		Penalty (Rp)	400,00
		Bonus (Rp)	0,00
		Total Project Bonus (Rp)	28.750,00

In the second step, the calculation was repeated/checked for the fixed and rounded variables (from in the first stage obtained continuous values to their nearest upper discrete values). Table 4 summarizes the optimum rounded solution for the small building project schedule.

The minimum total project cost obtained by the NLP optimization of the project schedule was found to be Rp.28,750.00 (in Billion). The gained optimal results include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities.

The example also shows that the total cost optimization of the project schedule performed by the NLP approach is carried out in a calculating process, where the start times and durations of project activities are considered simultaneously in order to obtain the minimum total project cost. The obtained maximum values for durations of the project activities 1, 2, 5, and6 demonstrate that the cost optimization of the project schedule not necessarily minimize the project duration. Moreover, the example shows that the optimum duration of the project may also exceed the target project duration. In this way, the additional feature of the total project cost optimization represents the advantage of the proposed NLP approach to construction project scheduling over the traditionally used CPM and PERT methods.

CONCLUSION

This paper presents the cost optimization of construction project schedules performed by the NLP approach. The NLP optimization model formulation for the cost optimization of construction project schedules was developed and applied. The input data within the NLP optimization model include: the project network with determined preceding and succeeding activities, the precedence relationships and the lag/lead times between activities, the normal/crash points and the direct cost-duration functions of the activities, the project indirect cost-duration function, the penalty-duration function and the bonus-duration function. The non-linear continuous total project cost objective function was subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. For specified input data, the NLP optimization yields the minimum total project cost and the construction project schedule with the optimal start times and the optimal durations of activities.

The existing exact NLP methods have focused on the cost optimal solution of the project scheduling problems which include simplifying assumptions regarding the precedence relationships among project activities. On the other hand, the present work aims to incorporate generalized precedence relationships between project activities and to propose the NLP model for making optimal project time-cost decisions applicable to actual construction projects. In addition, solving the construction project scheduling optimization problem using the proposed NLP model avoids the need for (piece-wise) linear approximation of the nonlinear expressions, which has been the traditional approach proposed for solving this optimization problem using the LP models. Since the proposed optimization approach enables an insight into the interdependence between the project duration and the total project cost, the decision-maker can more effectively estimate the effect of the project deadline on a total project cost before the submission of a tender

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