

# Inventory Simulation Optimization Under Non Stationary Demand

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## Abstract

The sales forecast is a prerequisite for the inventory decisions in the practical life. The right selection of the forecasting model to improve the inventory management can increase the customer service level and determine a better performance for the supply chain. The forecast performance should not only be evaluated by its standard error, but also its impact in the organization performance measures. The sales forecast should not be considered an individual function, but as an important part of supply chain management. By using simulation, managers are able to create a model of their supply chain systems and test various levels of input that can emulate real-life inconsistencies. We assume that we can model the supply chain as a network that each stage in the supply chain operates with a periodic review base-stock policy, that demand is bounded and that there is guaranteed service time between every stage and its customers. This study uses the analytic solver program in excel that does the randomization process on the variable demand.

**Keywords:** Nonstationary demand, Supply Chain, Inventory

## Introduction

Manufacturing firms are introducing new products at a higher frequency with increasingly shorter life cycles. For each new product, a firm must determine its supply chain and the associated operating policies to match supply to the demand to achieve the most revenue with the least cost. A major complication is that the demand process evolves over the product life cycle and is never stationary.

The product life cycle of a new product typically consists of four phases: (i) a product-launch phase in which the product is introduced to the market; (ii) a demand-ramp phase over which the demand rate grows rapidly; (iii) a peak-demand phase during which the product sells at its maximum rate; and (iv) an end-of-life phase during which the product demand declines as it is removed from the market.

The demand rate is never stationary because the product moves from one life-cycle phase to another. This research examines the problem of locating safety stocks in a supply chain in a way that accounts for uncertain, nonstationary

demand processes. Given the inherent complexity of modeling nonstationary demand processes, we seek a pragmatic approach that requires approximations and compromises to get results that might apply in practice. We use the modeling framework from Graves and Willems (2000) (referenced as G-W) and introduce a nonstationary demand model. We show that the G-W safety stock placement optimization applies to this case of nonstationary demand.

In the remainder of this section, we briefly discuss related literature. In §2, we present the key assumptions for modeling a supply chain and its nonstationary demand. In §3, we extend the G-W model to accommodate nonstationary demand. In §4, we examine a simple example to explore the near optimality of a constant service time (CST) policy.

**Related Literature.** Relative to the stationary-demand inventory literature, there is much less work for nonstationary demand. We characterize this work by how the nonstationary demand is specified and whether the work focuses on optimization versus performance evaluation. Morton and Pentico (1995) and Bollapragada and Morton (1999) focus on setting inventory policies for a single stage facing a general nonstationary demand process with proportional holding and backorder costs. When the order cost is zero, a time-varying base-stock policy is optimal; for a nonzero order cost, a time-varying policy is optimal. This research develops computationally efficient upper and lower bounds on these optimal policies. For short life cycle products, the challenge of accurately forecasting demand can be as important as determining inventory policies. Kurawarwala and Matsuo (1996) develop an integrated framework for forecasting and inventory management of short lifecycle products. Their approach estimates the parameters for a seasonal trend growth model and uses this as an input to a finite-horizon stochastic inventory model with time-dependent demands.

Nonstationary demand has also been modeled as a Markov-modulated Poisson demand process. One example is Chen and Song (2001), who show the optimality of echelon base-stock policies with state dependent order-up-to levels for

serial networks. A second example is Abhyankar and Graves (2001) who determine the optimal position of an inventory hedge in a two-stage serial supply chain that faces Markov modulated demand with two states. Within the bullwhip literature, several papers develop models for nonstationary demand.

Papers generally assume that each stage follows an adaptive base-stock policy and then analyze the effect that different forecasting techniques and assumed demand distributions have on the inventory requirements at each stage. For instance, Lee et al. (1997) demonstrate that the adjustment of order-up-to levels at the retailer amplifies the variance of the order signal the retailer provides for the manufacturer. Two other examples are Graves (1999) and Chen et al. (2000).

Finally, there is a growing body of work on designing supply chains to handle nonstationary demand. Beyer and Ward (2000) use simulation to accurately model the inventory requirements in a two-echelon supply chain that utilizes two modes of distribution and is subject to nonstationary demand. Johnson and Anderson (2000) investigate the benefits of postponement in supply chains that introduce multiple products with short product life cycles. Ettl et al. (2000) minimize the total inventory in a multistage inventory system, where the key challenge is to approximate the replenishment lead times within the supply chain. To model nonstationary demand, they break the horizon into a set of stationary phases and adopt a rolling-horizon approach where the optimization is performed for each demand phase.

### Supply Chain Model

In this section, we present the multistage model of a supply chain and the optimization problem for determining safety stocks. Graves and Willem (2008) show the model (D is maximum demand in interval time (s,t):

$$g_j(s, t) = D_j(s, t) - \int_{\tau=s}^t \mu_j(\tau) d\tau. \quad (1)$$

**Inventory Model.** The single-stage inventory model serves as the building block for modeling a multistage supply chain. We adapt the standard model of Kimball (1988) (see also Simpson 1958) to the case of nonstationary demand.

We define the inbound service time  $SI_j$  as the time for stage  $j$  to get supplies from its immediate suppliers. In each period  $t$ , stage  $j$  places an order equal to  $\phi_{ij}d_j(t)$  on each upstream stage  $i$  for which  $\phi_{ij} > 0$ . The time for all orders to be delivered to stage  $j$  dictates when stage  $j$  can commence production to replenish its demand. This inbound service time is constrained by the maximum outbound service time from the upstream suppliers, i.e.,  $SI_{ij} \geq \max_{(i,j) \in A} (S_i)$ .

We assume we are given base stocks  $B_j(t)$  for each period  $t=1,2,\dots,H$ . For the stated assumptions, we can express the inventory at stage  $j$  at the end of period  $t$  as

$$I_j(t) = B_j(t) - d_j(t - SI_j - T_j, t - S_j), \quad (2)$$

where  $d_j(s, t) = 0$  for  $s \leq t \leq 0$  and  $d_j(s, t) = d_j(0, t)$  for  $s < 0 < t$ .

We defined the net replenishment time for stage  $j$  to be its replenishment time, net the stage's promised outbound service time, i.e.,  $SI_j + T_j - S_j$ . This net replenishment time determines the safety stock at stage  $j$ . We always set the outbound service times so that the net replenishment time is nonnegative.

The explanation for Equation (2) follows that for the case stationary or certainty demand. There are three transactions in period  $t$ : stage  $j$  completes the replenishment of its demand from period  $t - SI_j - T_j$ ; stage  $j$  fills its demand from period  $t - S_j$ ; and stage  $j$  receives an additional replenishment equal to  $\Delta B_j(t) = B_j(t) - B_j(t-1)$  so as to have the prescribed base-stock level. Hence, we can write an inventory balance equation:

$$I_j(t) = I_j(t-1) + d_j(t - SI_j - T_j) - d_j(t - S_j) + \Delta B_j(t). \quad (3)$$

We obtain (2) by applying (3) recursively and using the boundary condition  $I_j(0) = B_j(0)$ .

To derive Equation (2), we implicitly assume that we can always make the necessary adjustment  $\Delta B_j(t)$  to the base-stock level. This need not be the case when the base-stock level decreases and  $\Delta B_j(t) < 0$ ; in effect, we need to assume that  $d_j(t - SI_j - T) + \Delta B_j(t) \geq 0$  so that the replenishment in period  $t$  is nonnegative. We expect this will typically be the case, and we assume this to be true so as not to overly complicate the presentation. (We note that when  $d_j(t - SI_j - T) + \Delta B_j(t) < 0$ , then Equation (2) provides a lower bound on the actual inventory level.)

**Determination of Base Stock.** For stage  $j$  to provide 100% service to its customers, we require that  $I_j(t) \geq 0$ ; we see from Equation (2) that this requirement equates to

Because demand is bounded, we satisfy the above requirement with the least inventory by setting the base stock as

$$B_j(t) \geq d_j(t - SI_j - T_j, t - S_j). \quad (4)$$

Thus, the base-stock level in period  $t$  is the maximum possible demand over a time interval  $(t - SI_j - T_j, t - S_j)$  for which stage  $j$  filled its demand, but has yet to receive replenishments.

**Safety Stock Model.** We use Equations (2) and (4) to find the expected inventory level  $E[I_j(t)]$ :

$$E[I_j(t)] = D_j(t - SI_j - T_j, t - S_j) - \int_{\tau=t-SI_j-T}^{t-S_j} \mu_j(\tau) d\tau = g_j(t - SI_j - T_j, t - S_j). \quad (5)$$

The expected inventory represents the safety stock held at stage  $j$  and depends on the net replenishment time and the demand bound. We observe that stage  $j$  holds no safety stock whenever the net replenishment time is zero, i.e.,  $SI_j + T_j - S_j = 0$ .

The supply chain will also have a work-in-process or pipeline inventory. This inventory corresponds to the replenishment of customer demand plus the planned adjustments to the base-stock levels. If we fix the base-stock levels for the start and end of the planning horizon, then we can show that the work in process does not depend on the choice of service times, but only on the average demand rates and the lead times at each stage. Hence, in posing an optimization problem, we ignore work-in-process and only model safety stock.

**Multistage Model.** To model the multistage system, we use Equation (5) for each stage where inbound service time is a function of the outbound service times for the upstream stages. We then formulate an optimization problem to find the optimal service times for the planning horizon:

$$\begin{aligned}
 \mathbf{P} \quad & \text{Min} \sum_{t=1}^H \sum_{j=1}^N h_j E[I_j(t)] = \sum_{t=1}^H \sum_{j=1}^N h_j g_j(t - S_j - T_j, t - S_j) \\
 \text{subject to} \quad & S_j - S_i \leq T_j \quad \text{for } j=1, \dots, N \\
 & S_j - S_i \geq 0 \quad \forall (i, j) \in A \\
 & S_j = 0 \quad \forall \text{ demand nodes } j \\
 & S_j, S_i \geq 0, \text{ integer for } j=1, \dots, N,
 \end{aligned}$$

where  $h_j$  denotes the holding cost per unit per time period for inventory at stage  $j$ . The objective of problem P is to minimize the safety stock holding cost over the planning horizon. The constraints assure that the net replenishment times are nonnegative, that each stage's inbound service time is no less than the maximum outbound service time quoted to the stage, and that the end-item stages satisfy their service guarantee.3

We define the planning model to end in period  $H$  and do not explicitly include any costs beyond this horizon. For instance, if  $H$  represents the end of the life cycle for a product, there would be disposal costs for the supply chain inventory left over at the end of the horizon. We could include a disposal cost by restating P with time-dependent holding costs.

To solve P, we first observe that we can rewrite the objective function as

$$\sum_{t=1}^H \sum_{j=1}^N h_j g_j(t - S_j - T_j, t - S_j) = H \times \sum_{j=1}^N h_j G_j(S_j, S_j),$$

where  $G_j(S_j, S_j) = H^{-1} \times \sum_{t=1}^H g_j(t - S_j - T_j, t - S_j)$  is the average safety stock at node  $j$  as a function of its inbound and outbound service times. Thus, P is equivalent to the safety stock optimization problem for stationary demand in G-W, but with its objective function expressed in terms of the average safety stock  $G_j(S_j, S_j)$  over the planning horizon  $H$ . Furthermore,  $G_j(S_j, S_j)$  is a concave function, given the assumption that the net demand bound  $g_j(s, t)$  is a concave function. As a consequence, we can solve P with the existing algorithms for stationary demand. G-W presents a dynamic programming algorithm for solving P for supply chains modeled as spanning trees; Humair and Willems (2006), Magnanti et al.

(2006), and Lesnaia (2004) have each developed and tested algorithms for general acyclic networks.

## Numerical Results

Four steps to convert an optimization model to a simulation optimization model:

### Step One:

Identify which inputs into the model that want to make uncertain:

- a. Choose which input cells to make uncertain. For this example let's assume we want to make the product demand in cells C14:E16 uncertain. For this example we'll assume a normal distribution.

As a result we'll make the following changes:

**Table 1:** Distribution of Variables

From		To	
Cell	Current Value	Cell	New Value
D14	50	D14	=INT(PsiNormal(50,5))
C15	20	C15	=INT(PsiNormal(20,2))
E15	100	E15	=INT(PsiNormal(100,10))
D16	50	D16	=INT(PsiNormal(50,5))

### Notes:

While a PsiNormal continuous distribution may be the best choice, demand should be integers so we use "Int()" to round the result.

For PsiNormal 50 represents our best guess at the mean value and 5 our best guess for the standard deviation.

In this model only some of the products had demand shown for a given time period so we left the other cells with a "0".

### Step two

Change the ending inventory  $\geq$  req. ending inventory constraints from Normal to Chance constraints:

Since the ending inventory levels are now dependent on the uncertain demand, we need to modify how they are captured in the model.

a. For each ending inventory constraint, click on it in the Model View of the Task Pane and in the Change Constraint dialog change Normal to VaR

b. At the same time, in the Chance field below where we chose VaR, enter 0.95

As a result we'll make the following changes:

**From:** The three ending inventory constraints (Ending\_Inventory\_T1  $\geq$  ending\_Invent\_Req\_T1, etc.) being Normal Constraints

**To:** The three ending inventory constraints (Ending\_Inventory\_T1, etc) being listed under Chance constraints and showing as VaR<sub>0.95</sub>(Ending\_Inventory\_T1)  $\geq$  Ending\_Invent\_Req\_T1 (etc.)

**Notes:**

We entered 0.95 because we want to meet our required ending inventory levels 95% of the time. Remember, our demand is uncertain so our ending inventory levels are also uncertain.

We could go with a lower or higher value depending on our flexibility.

**Step three**

*Capture the Mean result across the simulations and set our Objective cell as an Output cell:*

a. Choose where we want to show the Mean result across the simulations for our objective cell. We recommend placing it close to/next to the objective cell. We'll choose H43.

b. Enter the PsiMean function in that cell and point to your objective cell.

*As a result we'll make the following changes:*

**Table 2: Uncertainty Variable**

From		To	
Cell	Current Value	Cell	New Value
H43	Blank	H43	=PsiMean(H42)

**Notes:**

By adding in uncertainty into our model we are no longer solving for a single value in our Objective cell (in this case H42) but insight into the range of potential outcomes.

As a result, we want to see the mean value across the simulations and be able to see the range of outcomes so we can analyze results more in depth.

We do this by using the =PsiMean() function in one cell and point it to our objective cell (H42).

This also let's us double click that Objective cell (H42) and see the range of potential outcomes after we run the simulation optimization.

Alternatively, we could have added "+PsiOutput()" to H42 so it would read "=Sum(H28:H40)+PsiOutput". This would allow us to double click to see the range of results.

**Step four**

*Change the Objective cell in the model from Normal to Expected by double clicking on it in the Task Pane:*

a. If we look in the Task Pane with the Model tab selected you'll see at the top under Objective: Total\_Cost (Min).

b. Select this by double clicking on it and change Normal in the drop down to Expected.

*As a result we'll make the following changes:*

**From:** Total\_Cost (Min)

**To:** Expected(Total\_Cost)(Min)

**Notes:**

As mentioned above, our final result is no longer a single value in our Objective cell but a range of potential outcomes. Because of this we need to set our Objective cell to be an Expected value since the cost is dependent on uncertainty.

**Table 3: Final Test Inventory**

**Inputs To the Model**

<i>Initial Inventory</i>	Product 1	Product 2	Product 3	<i>Ordering Costs</i>	Product 1	Product 2	Product 3		
	Period 1	20	10		15	Each period	\$500	\$400	\$300
<i>Demand</i>	Product 1	Product 2	Product 3	<i>Holding Costs</i>	Product 1	Product 2	Product 3		
	Period 1	0	53		0	Each period per ending unit	\$10	\$10	\$10
	Period 2	19	0		85				
	Period 3	0	54		0				
<i>Req. Ending Inv.</i>	Product 1	Product 2	Product 3	<b>Constraint Values</b>					
	Period 1	0	20	0	Maximum purchases per product per period:			150	
	Period 2	0	20	0	Maximum total purchases per period:			500	
	Period 3	50	20	100	Minimum total purchases per period:			0	
				Amounts purchased must be in integers					
				Ending inventory must be >= to required ending inventory					

**Model**

		Product 1	Product 2	Product 3	Total	Inventory Cost	Order Cost
Initial Inventory		20	10	15			
Period 1	Purchase	0	0	0	0		\$0
	Demand	0	53	0			
	Ending Inventory	20	(43)	15	(8)	\$350	
Period 2	Purchase	0	0	0	0		\$0
	Demand	17	0	114			
	Ending Inventory	3	(43)	(99)	(139)	\$30	
Period 3	Purchase	0	0	0	0		\$0
	Demand	0	42	0			
	Ending Inventory	3	(85)	(99)		\$30	
						\$380	\$0
							\$380
						<b>Mean Cost:</b>	<b>#N/A</b>

**Conclusion**

In this study, it has been reviewed a model of placement safety stock in a supply chain with stationary demand and demonstrated how to extend supply chain model with demand stationary to obtain optimal safety stock placement in particular in short distribution products.

From ten simulations that have been done in this study, it was found the total cost is minimal, namely the simulation 4th with a total cost of 3,035 dollar, which amounted to 3981 units total demand and total supply of 7339 units. Means the initial stock of goods stored the remainder of last year by  $7339 - 3981 = 3358$  units. The total cost of storage for 1,835 dollars and the total cost booking of 1,200 dollars. The more simulations performed so possible total cost of the smaller will be obtained. It relies on randomization demand, initial stock, and receipt of goods.

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