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An Analytic Study of Dependence Claims Impact for Characteristics of Moment

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Abstract.

Risk model of aggregate claims usually was assumed that the value of claims between events is mutually independent. But there are possible cases that there is dependence between the value of the claims. This article discusses some of the characteristics of moment for collective risk model where the value of the claims between events has dependence. I used the Pearson Correlation for the depences of the claims. Results obtained that there was no impact between the characteristics of the moment caused by dependences for

$X_i \sim Binomial(m, \theta), m = 1,2,3, i = 1,2, N \; and \; NPOI(\lambda).$

Keywords: Dependence Claims, Moment Characteristic**.**

INTRODUCTION

Risk connotes negatively and is often translated as loss. The risk is uncertain. Risk measurements can be stochastic. The risk model is one way to translate the phenomenon of loss through distribution of statistics. Risk modeling can be used to predict risk in times that will come (forecasting future risk). That is, understanding the concept of stochastic processes (series time) is important.

One area closely related to risk is insurance. It happens because with insurance products is the transfer risk from the policy holder to the insurance committee. In claim loss modeling, there are two important measures that must be considered namely the frequency of claim loss and the magnitude or value or severity of claim loss.

Random losses and their distribution can be studied further through the nature of moments (especially up to moment 4) and tail distribution behavior. These two traits can be used as indicators the presence of extreme observations that are important in calculating risk. Insurance activities are basically related to losses (claims), both frequency and value or severity. The frequency of claims can be assessed through discrete random losses, in particular Poisson and binomial distributions.

Goovaerts-Dhaene [5] had explained that The correlation order, which is defined as a partial order between bivariate distributions with equal marginals, is shown to be a helpfull tool for deriving results concerning the riskness of portfolios with pairwise dependencies. Given the distribution functions of the individual risks, it is investigated how changing the dependency assumption influences the stop-loss premiums of such portfolios. Consider the individual risk theory model with the total claims of the portfolio during some reference period (e.g. one year) given by $S = \sum_{i=1}^{n} X_i$where X_i is the claim amount caused by policy$i$ $(i = 1,2, \cdots, n)$. In the sequel assumed that the individual claim amounts$X_i$are nonnegative random variables and that the distribution functions F_i of X_i are given. Usually, it is assumed that the risks X, are mutually independent because models without this restriction turn out to be less manageable. They derived results concerning the aggregate claims S if the assumption of mutually independence is relaxed. More precisely, They assumed that the portfolio contains a number of couples (e.g. wife and husband) with non-independent risks.

Hisakado, et al[5] explained a general method to construct correlated binomial distributions by imposing several consistent relations on the joint probability function. Hisakado, et al [5]obtained self-consistency relations for the conditional correlations and conditional probabilities. The beta-binomial distribution is derived by a strong symmetric assumption on the conditional correlations. Their derivation clarifies the 'correlation' structure of the beta-binomial distribution. It is also possible to study the correlation structures of other probability distributions of exchangeable (homogeneous) correlated Bernoulli random variables. They studied some distribution functions and discuss their behaviors in terms of their correlation structures**.**

In this paper, Iwill change the distribution. I use compound distribution between binomial and poisson distribution. Then i use the same correlation equation to get the moment or characteristic form of the compound distribution commonly.

CLAIM AGGREGATE DISTRIBUTIONS

Poisson Distribution

Analogous to the Bernoulli experiment, experiments or Poisson processes will examine.

(i) a lot of success in a time period, and

(ii) the (continuous) time required to obtain the first success.

The distribution involved in (i) is the Poisson distribution, whereas the distribution corresponding to (ii) is an exponential distribution.

Let N be a random loss expressing the frequency of the claim loss (which entered or filled) at a time period. The distribution for N is Poisson with parameters λ. The typical distribution is the same mean and variance λ,

$$E(N) = Var(N) = \lambda.$$

If there is a random Poisson loss (or other discrete random loss) then i can determining:

(i) the probability of loss frequency through an opportunity function or generating function opportunity or moment generating function

(ii) expected (conditional) frequency of loss.

Binomial Distribution

If a random loss N denotes the frequency of claimed losses processed from all incoming claims. The exact distribution for N is the binomial distribution with the parameters m (frequency of incoming claims) and θ (chance of claim is processed). Notation: $N \sim B(m, \theta)$. Function the opportunity for N is

$$P(N = k) = C_k^m \theta^k (1-\theta)^{m-k}, k = 0,1,2,\cdots,m$$

The nature of the moment, or the r-th moment, can be determined by utilizing the opportunity function is

$$E(N^r) = \sum_{k=0}^{m} n^r P(N = k)$$

For $m = 1$, for example, $E(N) = m\theta$ is obtained.

Compound Distribution

If $X_1, \cdots, X_n$ random sample of X with F_X distribution function. If N random variables are integer values that are independent of $X_1, \cdots, X_N$, then the random variable

$$S_N = X_1 + \cdots + X_N$$

is said to have a compound distribution, if:

- Distribution N referred to as the first distribution (primary distribution), and distribution X is said to be a secondary distribution.
- Naming distribution: primary-secondarydis-tribution.
- Poisson compound distribution is the distribution with the first distribution is the distribution Poisson and any distribution for the second distribution.

In the model of collective risk, aggregate claims distribution is assumed to follow a compound distribution,

$$S = X_1 + \cdots + X_N \quad (1)$$

With N random variables stating claims frequency and $X_1, \cdots, X_N$ random variables, identical and mutually independent, said the severity of the claims [2].

Denuit, et al [1] explained that:

(1) First moment of S

$$E(S) = E(N)E(X) \quad (2)$$

(2) Second Moment of S

$$Var(S) = E(N)\, var(X) + Var(N)\big(E(X)\big)^2 \quad (3)$$

In the case of collective risk models which the value of claims between the incidence, and this dependence may affect the characteristics of the collective risk model. Furthermore, I show that how their dependence takes effect on the moment of aggregate claims.The dependence discussed in this article is assumed to be positive in the form of Pearson correlation, namely:

$$\rho Corr(X_1, X_j) = \frac{E(X_1X_j) - E(X_1)E(X_j)}{\sqrt{E(X_1)(1-E(X_1))E(X_j)(1-E(X_j))}} \quad (4)$$

Moment Characteristics of Claims Aggregate

For example, given aggregate (S)as the equation (1), but $X_1, X_2, \cdots, X_N$has identical distribution and dependence. In this case, $E(X_i) = E(X), Var(X_i) = Var(X), i = 1,2,3,\cdots,N$, and ρas the equation (4), so I get the first and second moments of S are:

(1) First Moment (Expectation) of$S$

$$E(S) = E(N)E(X) \quad (5)$$

(2) Second Moment of$S$

$$E(S^2) = E(N)Var(X)(1-\rho) + E(N^2)\left(\rho Var(X) + \big(E(X)\big)^2\right)$$

(3) Second Centered Moment (Variance) of$S$

$$Var(S) = E(N)Var(X_i)(1-\rho) + E(N^2)\big(\rho Var(X)\big)$$

Suppose the random variable defined,

$$S = \sum_{i=1}^{N} X_i$$

Where $X_1, X_2, \cdots, X_N$has identical distribution and dependence, and $N \sim POI(\lambda)$.I assume that $Corr(X_i, X_j) = \rho, \forall i \neq j;\ i,j = 1,2,3,\cdots,N is\ positive$. So the first and second moments of S are,

$$E(S) = E[E(S|N)] = E\left[\sum_{n=0}^{\infty} E(S|N=n)P(N=n)\right]$$
$$= E\left[\sum_{n=0}^{\infty} nE(X)P(N=n)\right]$$
$$= E\left[\sum_{n=0}^{\infty} nP(N=n)\right]E(X) = E(N)E(X)$$
$$E(S^2) = E\left[\sum_{n=0}^{\infty} nP(N=n)\right]E(X) = E(N)E(X)$$
$$= \sum_{n=0}^{\infty} E\left[(\sum_{i=1}^{n} X_i)^2\right] P(N=n)$$
$$= \sum_{n=0}^{\infty} E\left[(\sum_{i=1}^{n} X_i{}^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} X_iX_j\right] P(N=n)$$
$$= \sum_{n=0}^{\infty} \left[\sum_{i=1}^{n} E(X_i{}^2) + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} E(X_iX_j)\right] P(N=n)$$
$$= \sum_{n=0}^{\infty}\left[nE(X_i{}^2) + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}(Kov(X_iX_j) + E(X_i)E(X_j))\right]P(N=n)$$
$$= E(N)E(X^2) + \sum_{n=0}^{\infty}[n(n-1)(\rho Var(X) + (E(X)^2)] P(N=n)$$
$$= E(N)E(X^2) + \left(\rho Var(X) + \left(E(X)\right)^2\right) \sum_{n=0}^{\infty}\left(n(n-1)\right)P(N=n)$$
$$= (N)E(X^2) + (\rho Var(X) + \left(E(X)\right)^2(E(N^2) - E(N))$$
$$= E(N)E(X^2) + \rho Var(X)E(N^2) + (E(X))^2(E(N^2) - \rho Var(X)E(N))$$
$$= E(N)Var(X) - \rho E(N)Var(X) + E(N^2)(\rho Var(X) + (E(X))^2)$$
$$= E(N)\, Var(X)(1-\rho) + E(N^2)\left((\rho Var(X) + \left(E(X)\right)^2\right)$$
$$Var(S) = E(S^2) - \left(E(S)\right)^2$$
$$[E(N)Var(X)(1-\rho) + E(N^2)\left(\rho Var(X) + \left(E(X)\right)^2\right)] - \left(E(N)E(X)\right)^2$$
$$= E(N)Var(X)(1-\rho) + E(N^2)\left(\rho Var(X)\right) + Var(N)\left(E(X)\right)^2$$

RESULTS

If $X_1, X_2, \cdots, X_N$ has Binomial distribution, write $X_1 \sim Binomial(m,\theta), M = 1,2,3, I = 1,2,\cdots,N$ and $N \sim POI(\lambda)$. So the first and second moments of S are:

(1) For $m = 1$, obtained:

$$E(S) = E(N)E(X) = \lambda\theta$$
$$E(S^2) = E(N)Var(X)(1-\rho) + E(N^2)\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$= E(N)Var(X)(1-\rho) + Var(N) + E(N)\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$= \lambda\theta(1-\theta)(1-\rho) + (\lambda+\lambda^2)(\rho\theta(1-\theta) + \theta^2)$$
$$Var(S) = E(S^2) - \left(E(S)\right)^2$$
$$= \lambda\theta(1 + \lambda\rho - \lambda\theta\rho)$$

(2) For $m = 2$, obtained:

$$E(S) = E(N)E(X) = 2\lambda\theta$$
$$E(S^2) = E(N)Var(X)(1-\rho) + E(N)^2\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$= E(N)Var(X)(1-\rho) + Var(N) + E(N)^2\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$= 2\lambda\theta(1-\theta)(1-\rho) + (\lambda+\lambda^2)(2\rho\theta(1-\theta) + 4\theta^2)$$
$$Var(S) = E(S^2) - \left(E(S)\right)^2$$
$$= 2\lambda\theta(1 + \theta + \lambda\rho - \lambda\theta\rho)$$

(3) For m=3, obtained:

$$E(S) = E(N)E(X) = 3\lambda\theta$$
$$E(S^2) = E(N)Var(X)(1-\rho) + E(N^2)\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$= E(N)Var(X)(1-\rho) + Var(N) + E(N)^2\left(\rho Var(X) + \left(E(X)\right)^2\right)$$
$$Var(S) = E(S^2) - \left(E(S)\right)^2$$
$$= 3\lambda\theta(1 + 2\theta + \lambda\rho - \lambda\theta\rho)$$

CONCLUSION

Based on the equations (2) and (5) i found that expectations of S does not depend on the value of the correlation between the value of the claims. While based on the equation (6) and (7), it can be said that the second moment and variance of S is proportional to the value of the correlation between the value of the claim.

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